Mathematical Hair Growth Model

- A mathematical model is a description of a system using mathematical language. The process of developing a mathematical model is termed mathematical modeling (also written *modeling*).

Objective

Our goal is using mathematical equations, to describe the variation of growth of hairs after the transplantation and to determine how the growth depends upon the time, and the number of grafts.

Materials and methods

We used the linear mathematical model ,and the knowledge of differential equation to establish our growth model ,and to solve the equations.
If h (t) a function represents a population of hairs , t = time g = h / λ , λ is the ratio of grafts depicts the <<quality>> of extraction (λ = number of hairs / number of grafts).

We suppose that the number of grafts g is very high, and the population of hairs can be considered as continuous and differentiable function of time t , so we can accept that variation rate dh / dt is proportional to the population of transplanted grafts g.

The previous approach can be completed, taking into consideration many factors influence the growth (blood support, skin type, damage of skin etc).

We can improve our model accepting the linear mathematical model, so, the best approach could be:

$dh / dt = a_1 . g + a_2 . g^2 + a_3 . g^3 + \ldots + a_n . g^n$.
Where $a_{1,2,3,\ldots,n}$ constants.

Materials and methods

- For simplicity we ignore all the factors exponents higher or equal to three, so, the total formula is :

$dh/dt = a_1 \cdot g - a_2 \cdot g^2$,

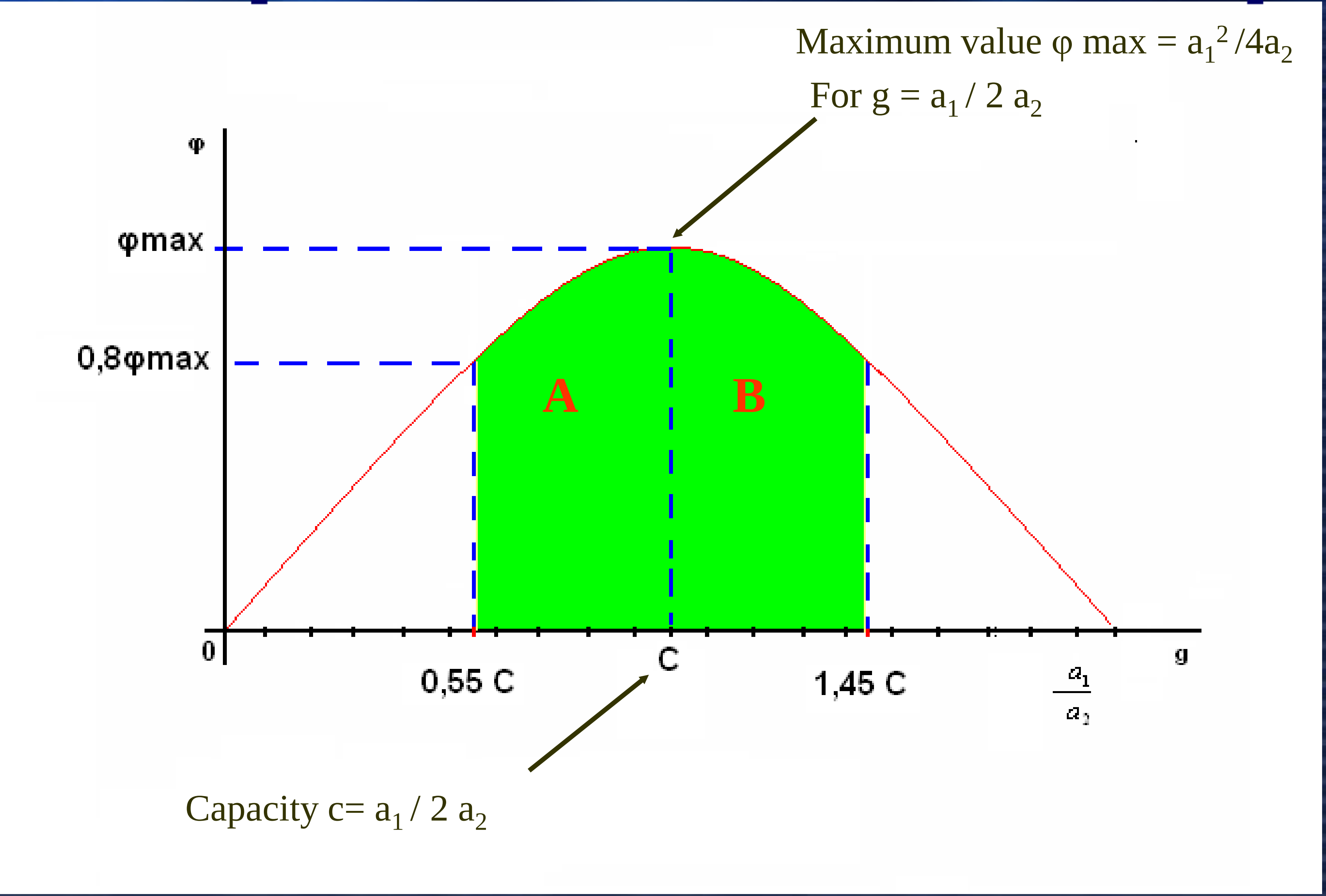
- Studying the above function we can achieve some very important equations
- Because $g = h / \lambda$ we have:
- $dh / dt = a_1 \cdot (h / \lambda) - a_2 (h / \lambda)^2$ (1)
- This is a differential equation and its solution is :

$$h(t) = \frac{a_1 \bullet \lambda}{a_2 (1 + c e^{-\frac{a_1 t}{\lambda}})}$$

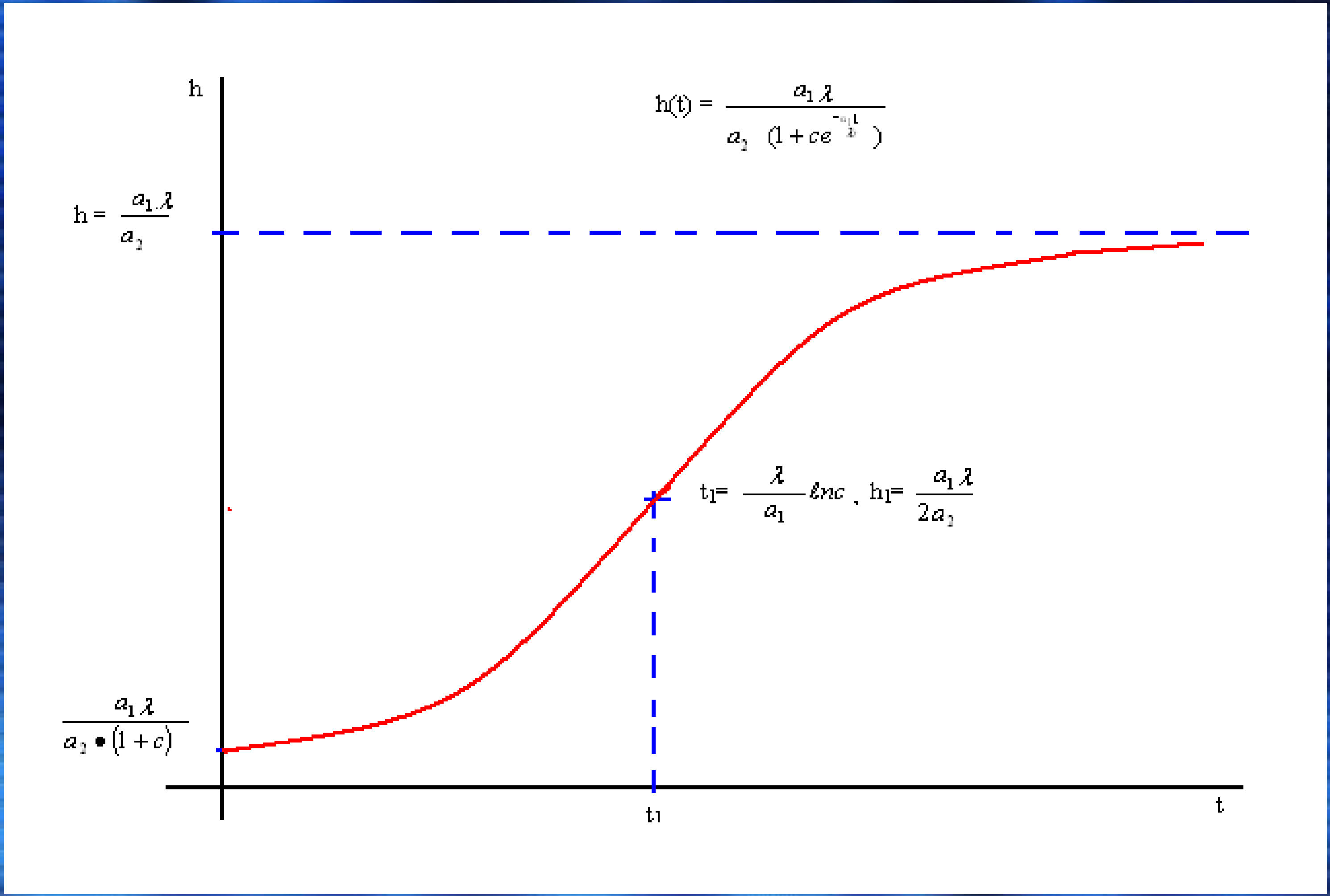
Materials and methods

- Secondly $\varphi = a_1 . g - a_2 . g^2$ (2)
- Where the first equation (1) represents the total number of hairs during time and equation (2) φ represents the growth and g is the number of grafts.

Depiction of function φ



Depiction of equation h (t)



Materials and methods

- The main idea of our model is that the growth that is the result of the hair operation depends upon 2 major parameters.
- One that influences with a positive way the development of the hairs and a second one that influences with negative way
- The interaction of the two important parameters may will determine the final result of the operation.

CONCLUSIONS

1. The growth depends upon the number of grafts
2. The maximum value is being determined for $g = a_1 / 2a_2$
3. After this value the growth is being decreased.
4. For $g = a_1 / a_2$ the growth is zero !!!!!

CONCLUSIONS

5. **Generally the growth depends upon biological characteristics of skin $a_1 a_2$. We can determine $a_1 a_2$ and the value $a_1 / 2a_2$ as**

THE CAPACITY OF SKIN

$$C = a_1 / 2a_2.$$

We conclude

a) There is an interval between 0,55 c up to 1,45 c where the growth:

$$0{,}8\ \varphi\ max \leq \varphi \leq \varphi\ max$$

b) we have the same growth both in two intervals, A and B

c) There is no reason to work in the interval B

CONCLUSIONS

6. Capacity reflects the number of hairs or grafts than can be placed safely in one session.
7. We believe that the value of capacity changes during the life in the same person
8. For t = 0 the value of hairs is not zero , that means some grafts start to grow immediately.

CONCLUSIONS

9. The rate of growth changes during the time , faster in the beginning , slower later.
10. The constants a_1 , a_2 reflects the physiological or biological characteristics of skin changing from person to person.
11. The maximum value of h (hairs) depends upon the λ. The final number of hairs is proportional to λ and depends on a_1 and a_2.
12. Theoretically the total number of hairs will grow after infinity time